SAMAGRA SHIKSHA KERALA

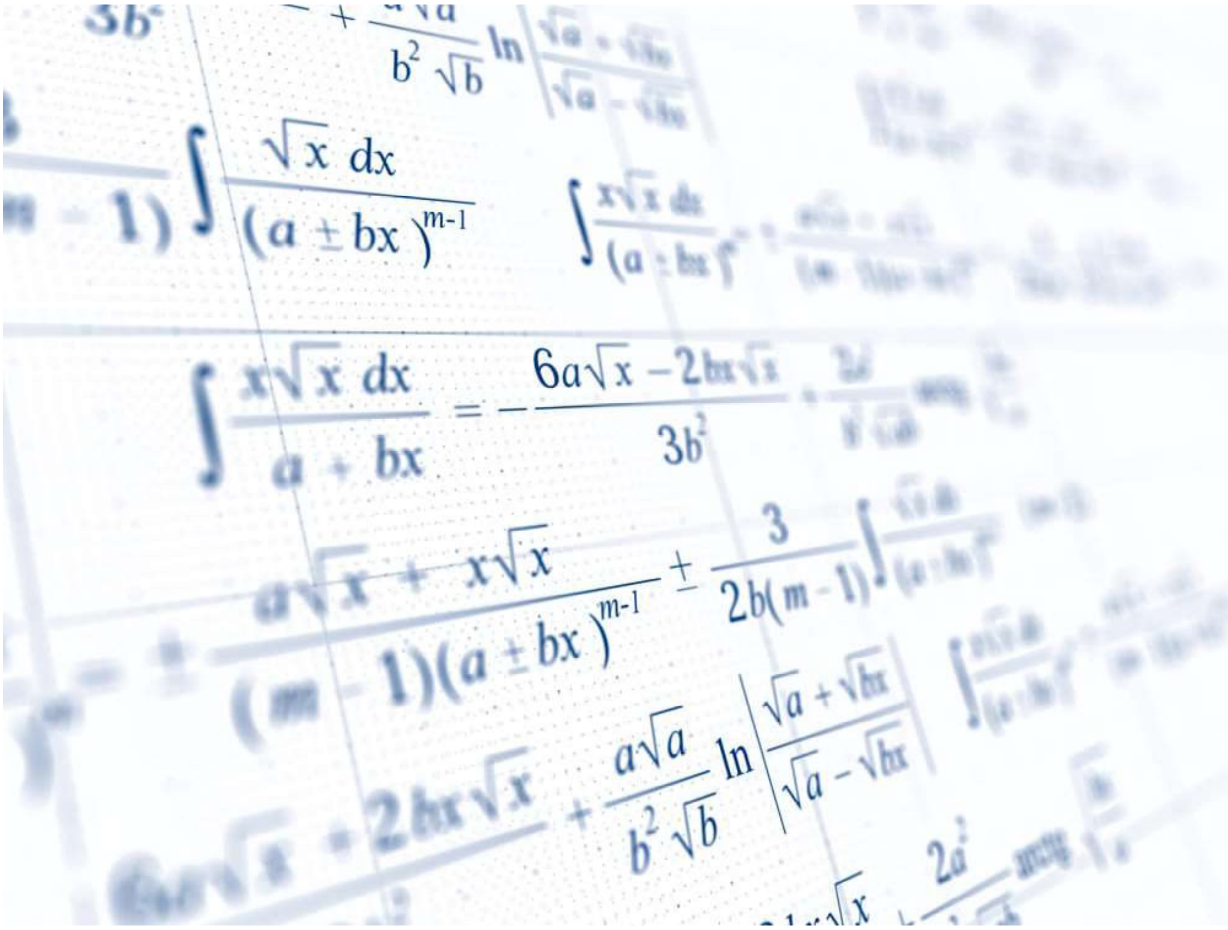
RIGHT TO EDUCATION

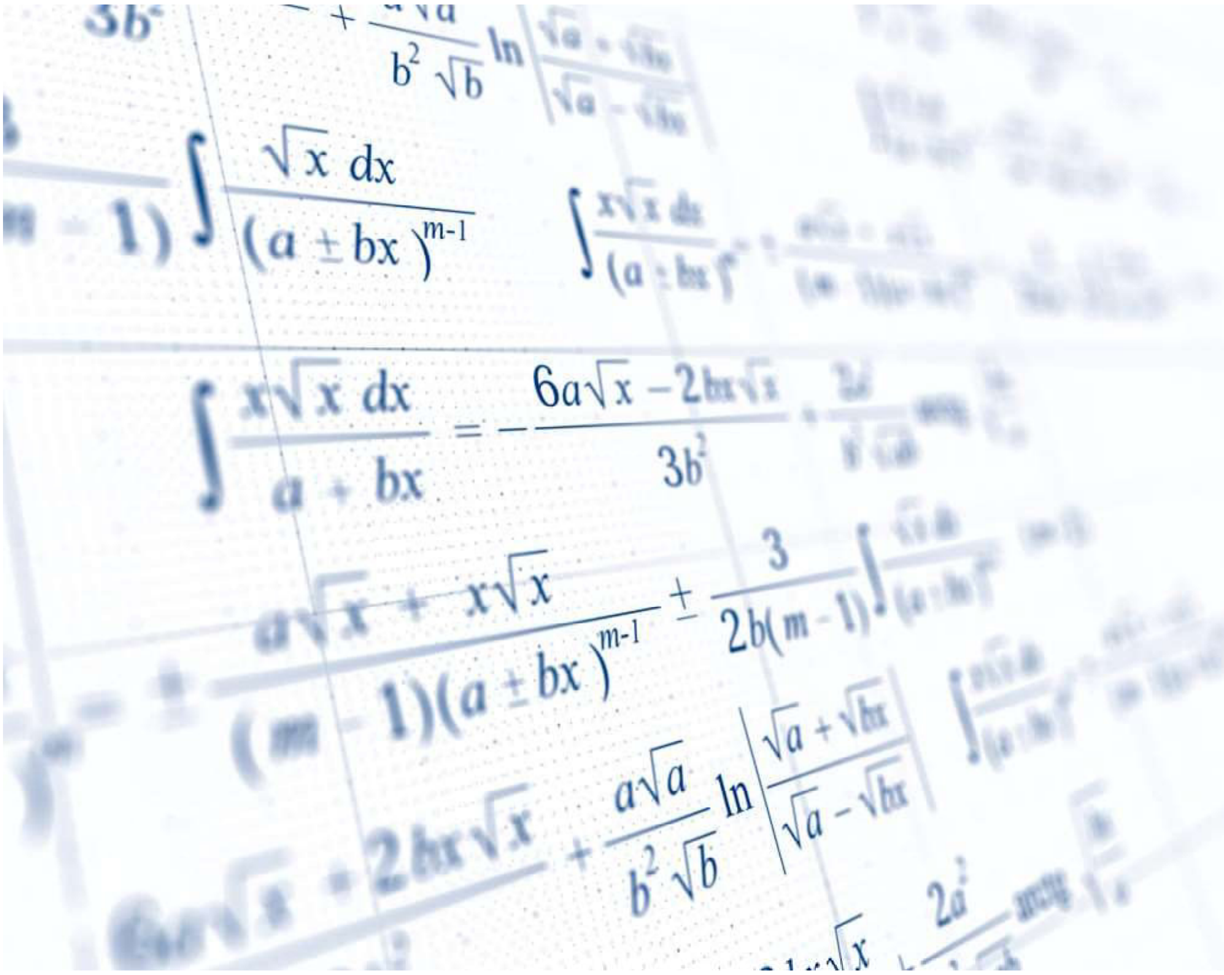
SAMAGRA SHIKSHA KERALA

Every one learns Every one Grows

Plus One
MATHEMATICS
Module

2023





Maths +1

1. Sets
2. Relations and Functions
3. Trigonometric Functions
4. Complex Numbers and Quadratic Equations
5. Linear Inequalities
6. Permutations and Combinations
7. Binomial Theorem
8. Sequences and Series
9. Straight Lines
10. Conic Sections
11. Introduction to Three Dimensional Geometry
12. Limits and Derivatives
13. Statistics
14. Probability

ആമുഖം

ഹയർസെക്കൻഡറി തലത്തിൽ ഒന്നാംവർഷ ഗണിതശാസ്ത്രപഠനം ലഘൂകരിക്കുന്നതിന് വേണ്ടിയുള്ള സമഗ്രവും ലളിതവുമായുള്ള ഒരു പഠനസഹായിയാണ് ഇത്. ഇതിൽ ഓരോ യൂണിറ്റിലേയും ഏറ്റവും പ്രധാന ആശയങ്ങളും അതുമായി ബന്ധപ്പെട്ട ചോദ്യങ്ങളും ഉത്തരങ്ങളും ഈ ബുക്ക്‌ലെറ്റിൽ ഉൾപ്പെടുത്തിയിട്ടുണ്ട്.

ആശയങ്ങളും സൂത്രവാക്യങ്ങളും 'Gold Coins' എന്നും ചോദ്യങ്ങൾ 'Golden Problems' എന്നുമുള്ള ശീർഷകങ്ങളിൽ ആണ് അവതരിപ്പിച്ചിട്ടുണ്ട്.

സ്റ്റേറ്റ് പ്രോജക്ട് ഡയറക്ടർ

Chapter 1

SETS

GOLD COINS

- If a set having 'n' elements, then
  a) Number of subsets : 2^n
  b) Number of proper subsets : $2^n - 1$
- $(a,b)=\{x:x\in R, a<x<b\}$
- $[a,b)=\{x:x\in R, a\le x<b\}$
- $(a,b]=\{x:x\in R, a<x\le b\}$
- $[a,b]=\{x:x\in R, a\le x\le b\}$
- $A\cup B=\{x:x\in A \text{ or } x\in B\}$
- $A\cap B=\{x:x\in A \text{ and } x\in B\}$
- $A-B=\{x:x\in A \text{ and } x\notin B\}$
- $A'=\{x:x\in \cup \text{ and } x\notin A\}$
- $A\cup A'=\cup$
- $A\cap A'=\phi$
- $(A\cap B)'=A'\cup B'$
- $(A\cup B)'=A'\cap B'$
- $U\cup A=U$, $U\cap A=A$
- $(A')'=A$
- If $A\subset B$ them $A\cup B=B$ and $A\cap B=A$
- Two sets A and B are disjoint, then $A\cap B=\phi$.

Golden Problems

1. Write the following sets in roster form.

   a) A = { x : x is an integer and -3 < x < 7 }

   b) B = {x : x is a prime number <10}

   c) $C = \left\{ x : x \text{ is an integer, } -\frac{1}{2} < x < \frac{9}{2} \right\}$

   d) $D = \left\{ x : x \text{ is an integer, } x^2 \leq 4 \right\}$

2. Write all the subsets of the set A = { 1, 2, 3 }.

3. Choose the correct answer.

   a) Which one of the following is equal to $\{x : x \in R, 3 < x \leq 5\}$

      i) [3, 5] ii) [3, 5) iii) (3, 5) iv) (3, 5]

   b) If A and B are two sets such that $A \subset B$ then $A \cup B$ is

      i) A ii) B iii) U iv) $\varnothing$

   c) If U is the Universal set and A is any set then $U \cap A =$ ....

      i) U ii) A iii) $\varnothing$ iv) A'

4. Let A={2, 3, 4, 5} and B = {4, 5, 6, 7}

   a) Write $A \cup B$

   b) Write $A \cap B$

   c) Write A-B

   d) Write B-A

   e) Verify that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

5. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$

$B = \{2, 3, 5, 7\}$

Find

a) A' and B'

b) $A \cup B$

c) $A \cap B$

d) Verify that $(A \cup B)' = A' \cap B'$

e) Verify that $(A \cap B)' = A' \cup B'$

Solutions

1. a) $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

   b) $B = \{ 2, 3, 5, 7 \}$

   c) $C = \{ 0, 1, 2, 3, 4 \}$

   d) $D = \{ -2, -1, 0, 1, 2 \}$

2. Subsets : $\{1\}, \{2\}, \{3\}, \{ 1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \varnothing$

3. a) iv) $(3, 5]$

   b) ii) B

   c) ii) A

4. $A = \{2, 3, 4, 5\}$

   $B = \{4, 5, 6, 7\}$

   a) $A \cup B = \{2, 3, 4, 5, 6, 7\}$

   b) $A \cap B = \{4, 5\}$

   c) $A - B = \{2, 3\}$

   d) $B - A = \{6, 7\}$

   e) $(A \cup B) - (A \cap B) = \{2, 3, 4, 5, 6, 7\} - \{4, 5\}$

   $= \{2, 3, 6, 7\}$

   $(A - B) \cup (B - A) = \{2, 3\} \cup \{6, 7\}$

   $= \{2, 3, 6, 7\}$

   $\therefore (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

5. U={1, 2, 3, 4, 5, 6, 7, 8, 9}

A={2, 4, 6, 8}

B={2, 3, 5, 7}

a) $A' = U - A = \{1,3,5,7,9\}$

$B' = U - B = \{1,4,6,8,9\}$

b) $AUB = \{2,3,4,5,6,7,8\}$

c) $A \cap B = \{2\}$

d) $(A \cup B)' = U - (A \cup B) = \{1,9\}$

$A' \cap B' = \{1,9\}$

$\therefore (A \cup B)' = A' \cap B'$

e) $(A \cap B)' = U - (A \cap B) = \{1,3,4,5,6,7,8,9\}$

$A' \cup B' = \{1,3,4,5,6,7,8,9\}$

$\therefore (A \cap B)' = A' \cup B'$

Chapter 2
RELATIONS AND FUNCTIONS

GOLD COINS

- For any two non - empty sets A & B

  $A \times B = \{(a, b) : a \in A, b \in B\}$

- If n(A) = p and n (B) = q, then $n(A \times B) = pq$
- If (a,b) = (c,d), then a = c and b = d
- If n (A) = P, n (B) = q, then total number of relations from A to B is 2^{pq}
- If R is a relation from A to B then

  Codomain = B

  Domain = $\{x \in A : (x, y) \in R\}$

  Set of all first elements in R

  Range = $\{y \in B : (x, y) \in R\}$

  = Set of all second elements in R.

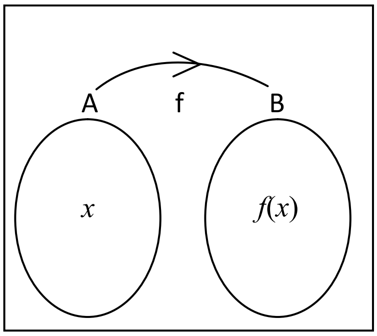
- A function f: A $\rightarrow$B is a relation in which each element of A has one and only one image in B.
- If f : A $\rightarrow$B is a function then

  Domain = A

  Codomain = B

  Range = $\{f(x), x \in A\}$

  = Set of Images



- If f : X $\rightarrow$R and g : X $\rightarrow$R then

  $(f+g)x = f(x) + g(x), x \in X$

  $(f-g)x = f(x) - g(x), x \in X$

  $(f.g)x = f(x) . g(x)$

  $(k.f)x = k(f(x))$, k is a constant

  $\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)}, g(x) \neq 0$

Golden Problems

1. If A={2, 3} and B = {3, 4, 5} then show that $A \times B \neq B \times A$

2. If A={-1, 1} then find $A \times A \times A$

3. Find the values of x and y if $\left(\frac{x}{2}, \frac{y}{3}+1\right) = \left(2, \frac{1}{3}\right)$

4. Let A={1, 2, 3, ...14} Define a relation from A to A by R=$\{(x, y) : 2x-y = 0, x,y \in A\}$
   a) Write R in roster form
   b) Find domain, codomain and range of R

5. If A = {2, 3} , B = {1, 3, 5} then the number of relations from A to B is ....
   i) 2 ii) 32 iii) 64 iv) 62

6. Define signum function, write its domain and range. Also draw its graph.

7. Draw the graph of the function f(x)=$|x+1|$.

8. Find the domain of f(x) = $\frac{x^2+3x+5}{x^2-5x+4}$

9. Consider the relation R={(2,1), (3,4), (4,5)}
   State whether R is a function or not.
   If it is a function write its domain and range.

10. Let $f : R \rightarrow R$ and $g : R \rightarrow R$ are two functions defined by $f(x) = x^2$, $g(x) = 2x+1$. Find.
    a) $f+g$
    b) $f.g$
    c) $\frac{f}{g}$

Solutions

1. $A \times B = \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$

   $B \times A = \{3, 2), (3, 3), (4, 2), (4, 3), (5, 2), (5, 3)\}$

   $\therefore A \times B \neq B \times A$

2. $A \times A = \{-1, 1\} \times \{-1, 1\}$

   $= \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

   $A \times A \times A = \{(-1, -1, -1), (-1, 1, -1), (1, -1, -1), (1, 1, -1), (-1, -1, 1), (-1, 1, 1), (1, -1, 1), (1, 1, 1)\}$

3. $\frac{x}{2} = 2 \qquad \frac{y}{3} + 1 = \frac{1}{3}$

   $x = 4 \qquad 3 + y = 1$

   $y = 1-3$

   $= -2$

4 $2x - y = 0 \Rightarrow y = 2x$

   a) R = {(1,2), (2,4), (3,6), (4,8), (5,10), (6,12), (7,14)}

   b) Domain = {1, 2, 3, 4, 5, 6, 7}

   Codomain = A

   Range = {2, 4, 6, 8, 10, 12, 14}

5. Number of relations $= 2^{2\times 3}$

   $= 2^6$

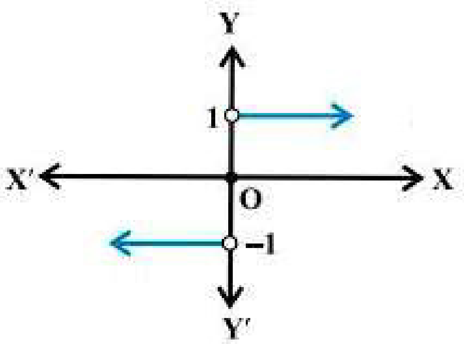
   $= 64$

6. $f : R \to R$ defined by

$$f(x) = \begin{cases} 1, & if \quad x > 0 \\ 0, & if \quad x = 0 \\ -1, & if \quad x < 0 \end{cases}$$

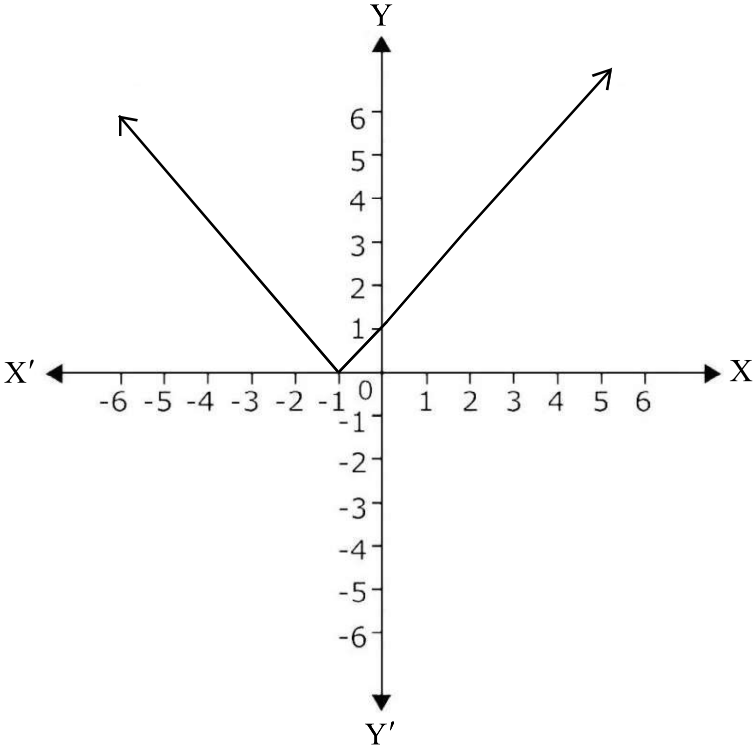
Domain = R

Range = {-1, 0, 1}



7.

x	-4	-3	-2	-1	1	2	3	4
f(x)=$\lvert x+1 \rvert$	3	2	1	0	2	3	4	5



8. x^2-5x+4 =0

$\Rightarrow$ (x-1)(x-4) =0

$\Rightarrow$ x=1, 4

Domain = R-{1,4}

9. The given relation is a function since every element in the first set has only one image in the second set.

Domain = {2, 3, 4}

Range = {1, 4, 5}

10. $f(x) = x^2$, $g(x) = 2x+1$

a) $(f+g)(x) = f(x)+g(x)$

$= x^2+2x+1$

b) $(f.g)(x) = f(x).g(x)$

$= x^2(2x+1)$

c) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{2x+1}, x \neq \frac{-1}{2}$

Chapter 3

TRIGONOMETRIC FUNCTIONS

GOLD COINS

- 1 Radian = $\frac{180}{\pi}$ degree
- 1 Degree = $\frac{\pi}{180}$ radian
- In a circle of radius r, an arc length ℓ substends an angle θ radian then

  $\ell = r\theta$
- $\sin^2 x + \cos^2 x = 1$

  $\sec^2 x - \tan^2 x = 1$

  $\operatorname{cosec}^2 x - \cot^2 x = 1$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$

  $\sin(x - y) = \sin x \cos y - \cos x \sin y$

  $\cos(x+y) = \cos x \cos y - \sin x \sin y$

  $\cos(x-y) = \cos x \cos y + \sin x \sin y$

  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

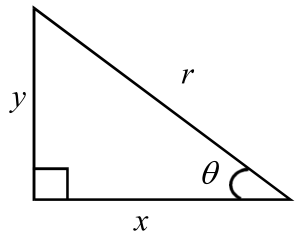
  $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

  $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

  $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

  $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\sin 2x = 2\sin x \cos x$

  $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

- 

$\operatorname{Sin}\theta = \frac{y}{r}$ $\qquad$ $\operatorname{Cos}ec\theta = \frac{r}{y}$

$\operatorname{Cos}\theta = \frac{x}{r}$ $\qquad$ $\operatorname{Sec}\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad$ $\operatorname{Cot}\theta = \frac{x}{y}$

- **II** **S** — Sin θ and Cosec θ are positive

  I **A** — All Trigonomeric functions are positive

  III **T** — Tan θ and Cot θ are positive

  IV **C** — Cos θ and Sec θ are positive

	0	30^0	45^0	60^0	90^0
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Golden Problems

1. Convert 120^0 in to radian measure.

2. Convert $\frac{5\pi}{6}$ rad into degree measure .

3. If $\text{Sin}\, x = \frac{-3}{5}, x$ lies in 3rd quadrant. Find the other five trigonometric functions.

4. Find sin 15^0

5. Prove that $\frac{\text{Sin}\, 5x + \text{Sin}\, 3x}{\text{Cos}\, 5x + \text{Cos}\, 3x} = \tan 4x$

6. Show that tan x. tan $2x$. tan $3x$ = tan $3x$ - tan$2x$ - tan$x$

7. a) Prove that $\frac{\text{Sin}\, x + \text{Cos}\, x}{\text{Sin}\, x - \text{Cos}\, x} = \frac{\tan x + 1}{\tan x - 1}$

   b) If tan$x$=$\frac{3}{4}$. Find the value of $\frac{\text{Sin}\, x + \text{Cos}\, x}{\text{Sin}\, x - \text{Cos}\, x}$

8. Prove that Cos$4x$ =1-8Sin^2x Cos^2x

9. Prove that $\text{Cos}\left(\pi/4 + x\right) + \text{Cos}\left(\pi/4 - x\right) = \sqrt{2}\,\text{Cos}\, x$

Solutions

1. $120^0 = 120 \times \frac{\pi}{180}$

$= \frac{2\pi}{3}$ radian

$1^0 = \frac{\pi}{180}$ radian

2. $\frac{5\pi}{6} = \frac{5\pi}{6} \times \frac{180}{\pi}$

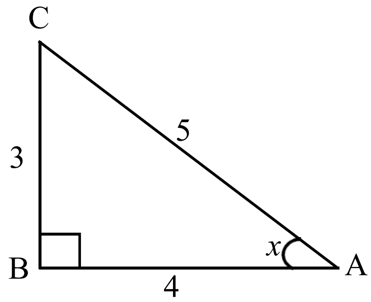
$= 150^\circ$

1 radian $= \frac{180^\circ}{\pi}$ degree

3. $\text{Sin } x = \frac{-3}{5}$ $\quad$ $\text{Cosec } x = \frac{-5}{3}$

$\text{Cos } x = \frac{-4}{5}$ $\quad$ $\text{Sec } x = \frac{-5}{4}$

$\tan x = \frac{3}{4}$ $\quad$ $\text{Cot } x = \frac{4}{3}$



$AB = \sqrt{5^2 - 3^2}$

$= \sqrt{16} = 4$

4. $\text{Sin } 15^0 = \text{Sin } (45\text{-}30)$

$= \text{Sin}45\text{Cos}30 - \text{Cos}45\text{Sin}30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Sin(x+y) = Sin$x$Cos$y$+Cos$x$Sin$y$

5. $\dfrac{\text{Sin}\,5x+\text{Sin}\,3x}{\text{Cos}\,5x+\text{Cos}\,3x}$

$\text{Sin}\,x+\text{Sin}\,y = 2\,\text{Sin}\dfrac{x+y}{2}\text{Cos}\dfrac{x-y}{2}$

$\text{Cos}\,x+\text{Cos}\,y = 2\,\text{Cos}\dfrac{x+y}{2}\text{Cos}\dfrac{x-y}{2}$

$$= \frac{2\,\text{Sin}\dfrac{5x+3x}{2}.\text{Cos}\dfrac{5x-3x}{2}}{2\,\text{Cos}\dfrac{5x+3x}{2}.\text{Cos}\dfrac{5x-3x}{2}}$$

$$= \frac{2\,\text{Sin}\,4x}{2\,\text{Cos}\,4x}$$

$= \tan 4x$

6. $\tan 3x = \tan(2x+x)$

$\tan 3x = \dfrac{\tan 2x+\tan x}{1-\tan 2x.\tan x}$

$\tan(x+y) = \dfrac{\tan x+\tan y}{1-\tan x.\tan y}$

$\tan 3x\,(1-\tan 2x.\tan x) = \tan 2x + \tan x$

$\tan 3x - \tan 3x.\tan 2x.\tan x = \tan 2x + \tan x$

$\therefore\ \tan 3x - \tan 2x - \tan x = \tan 3x.\tan 2x.\tan x$

7. a) $\dfrac{\text{Sin}\,x+\text{Cos}\,x}{\text{Sin}\,x-\text{Cos}\,x} = \dfrac{\dfrac{\text{Sin}\,x}{\text{Cos}\,x}+\dfrac{\text{Cos}\,x}{\text{Cos}\,x}}{\dfrac{\text{Sin}\,x}{\text{Cos}\,x}-\dfrac{\text{Cos}\,x}{\text{Cos}\,x}}$ [dividing by cos x]

$= \dfrac{\tan x+1}{\tan x-1}$

b) $\tan x = \dfrac{3}{4}$

$\dfrac{\text{Sin}\,x+\text{Cos}\,x}{\text{Sin}\,x-\text{Cos}\,x} = \dfrac{\tan x+1}{\tan x-1} = \dfrac{3/4+1}{3/4-1}$

$= \dfrac{7/4}{-1/4} = -7$

8. $\text{Cos}2x = 1-2\text{Sin}^2x$

$\text{Cos}4x = 1-2(2\text{Sin}x\ \text{Cos}x)^2$

$= 1-2(4\text{Sin}^2x.\text{Cos}^2x)$

$= 1-8\ \text{Sin}^2x\text{Cos}^2x$

9. $\text{Cos}\left(\pi/4+x\right)+\text{Cos}\left(\pi/4-x\right)$

$$= 2\,\text{Cos}\frac{\pi/4+x+\pi/4-x}{2}.\text{Cos}\frac{\left(\pi/4+x-\left(\pi/4-x\right)\right)}{2}$$

$$= 2\,\text{Cos}\frac{\frac{2\pi}{4}}{2}.\text{Cos}\frac{2x}{2}$$

$$\boxed{\text{Cos}\,x+\text{Cos}\,y = 2\,\text{Cos}\frac{x+y}{2}.\text{Cos}\frac{(x-y)}{2}}$$

$$= 2\,\text{Cos}\,\pi/4.\text{Cos}\,x$$

$$= 2.\frac{1}{\sqrt{2}}\text{Cos}\,x$$

$$\boxed{\text{Cos}\frac{\pi}{4}=\frac{1}{\sqrt{2}}}$$

$$=\sqrt{2}\,\text{Cos}\,x$$

Chapter 4

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

GOLD COINS

For the Complex Number $Z = a+ib$, then

- $Re(Z) = a$ and $Im(Z) = b$
- Conjugate of Z ($\bar{Z}$) = $a - ib$
- Modulus of Z ($|Z|$) = $\sqrt{a^2+b^2}$
- Multiplicative Inverse of Z (Z^{-1}) = $\dfrac{\bar{Z}}{|Z|^2}$
- $i^2 = -1$
- $i^{4n} = 1$

Golden Problems

1. Express the complex numbers $i^{9} + i^{14}$ in the form a + ib.
2. Express the complex number 3 (7+i7) + i(7+i7) in the form a+ib.
3. Express the complex number $\frac{3-2i}{1+2i}$ in the form a+ib.
4. Express the complex number (5+3i) (-2+i) in the form a+ib.
5. Express the complex number i^{-37} in the form a+ib.
6. Consider the complex number Z = 4-3i.
   (a) Find the conjugate of Z $(\overline{Z})$
   (b) Find the modulus of Z $(|Z|)$
   (e) Find the multiplicative inverse of Z (Z^{-1}).
7. Represent 2+3i in Argand plane.

Solutions

1. Let Z $= i^9 + i^{14}$

$= i^8 i + (i^2)^7$

$= (i^2)^4 i + (^-1)^7$

$= (^-1)^4\ i + ^-1$

$= i + ^-1 = ^-1 + i$

$\left[\because i^2 = -1\right]$

$(^-1)^{\text{odd number}} = ^-1$

$(^-1)^{\text{even number}} = 1$

2. Let Z $= 3\,(7+i7) + i\,(7+i7)$

$= 3\times 7 + i\times 3\times 7 + i\times 7 + i\times i7$

$= 21 + 21i + 7\,i + i^2 7$

$= 21 + 28i - 7$

$= 14 + i28$

3. Let Z $= \frac{3-2i}{1+2i}$

$= \frac{(3-2i)(1-2i)}{(1+2i)(1-2i)}$

$= \frac{3-6i-2i+4i^2}{1-4i^2}$

$= \frac{3-8i-4}{1+4} = \frac{-1+8i}{5}$

$= -\frac{1}{5} + i\frac{8}{5}$

4. Let Z $= (5+3i)\,(-2+i)$

$= 5\times {}^-2 + 5i + 3i\times {}^-2 + 3i\times i$

$= {}^-10 + 5i - 6i + 3i^2 = -10 + -i - 3$

$= {}^-13 - i$

5. Let Z $= i^{-37}$

$= \frac{1}{i^{37}}$

$= \frac{1}{i^{36} i}$

$= \frac{1}{(i^2)^{18}\, i}$

$\bar{x}^n = \frac{1}{x^n}$

$$= \frac{1}{(-1)^{18}\, i}$$

$$\therefore (-1)^{\text{even number}} = 1$$

$$= \frac{1}{i} = \frac{i}{i^2}$$

$= -i$

$= 0-i$

6. Let Z = 4 - 3i

a) $\bar{Z} = 4+3i$

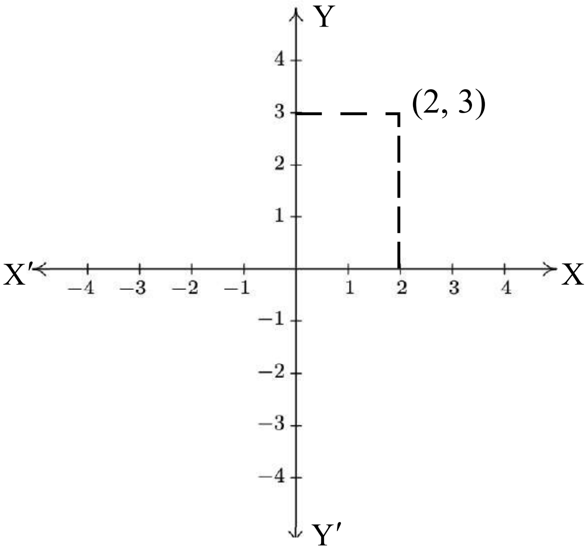
b) $|Z| = \sqrt{a^2+b^2}$

$= \sqrt{4^2+3^2} = \sqrt{25} = 5$

c) $Z^{-1} = \frac{\bar{Z}}{|Z|^2} = \frac{4+3i}{25}$

$= \frac{4}{25} + i\frac{3}{25}$

7.



Here X-axis = Real Axis.

Y-axis = Imaginary axis

Chapter 5

LINEAR INEQUALITIES

GOLD COINS

- Two real numbers or two algebraic expressions related by the symbol '<' , '>', ' $\leq$ ' , ' $\geq$ ' form an inequality.
- Equal numbers may be added to (or subtracted from) both sides of an inequality.
- Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.

Golden Problems

1. Solve $24x < 100$ when

   i) x is a natural number

   ii) x is a integer.

2. Solve $\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$

3. Solve $3x-2 < 2x+1$. Show the graph of the solutions on number line.

4. Solve $x+\frac{x}{2}+\frac{x}{3}<11$

5. Reghu obtained 68 and 72 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of atleast 60 marks.

6. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Solutions

1. $24x < 100$

$x < \frac{100}{24}$

$\because x < \frac{100}{24}$

ie. $x < 4.167$

i) When x is a natural number

Solution set of inequality = {1, 2, 3, 4}

ii) When x is an integer the solution

set of inequality = { ...., -3, -2, -1, 0, 1, 2, 3, 4 }

2. $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$3 \times 3(x-2) \leq 5 \times 5(2-x)$

$9(x-2) \leq 25(2-x)$

$9x - 18 \leq 50 - 25x$

$9x + 25x \leq 50 + 18$

$34x \leq 68$

$x \leq \frac{68}{34}$

$\therefore x \leq 2$

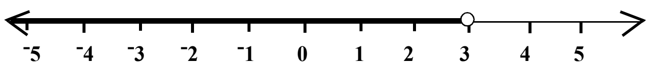
The solution set of the inequality is $x \in (-\infty, 2]$

3. $3x - 2 < 2x+1$

$3x - 2x < 1+2$

$x < 3$

The graphical representation of the solutions are



4. $x + \frac{x}{2} + \frac{x}{3} < 11$

Multiplying by 6

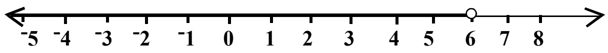
$6x + 3x + 2x < 66$

$11x < 66$

$x < \frac{66}{11}$

$x<6$

The graphical representation of the solutions are



5. Let x be the marks obtained by student in the third test. Then

$\frac{68+72+x}{3} \geq 60$

$\frac{140+x}{3} \geq 60$

$x \geq 180-140$

$x \geq 40$

Thus the student must obtain a minimum of 40 marks to get an average of at least 40 marks.

6. Let x , $x + 2$ be the odd positive integers. Then

$x < 10$ and $x + 2 < 10$..........................(1) [i.e., $x < 8$]

and $x + x + 2 > 11$

$2x + 2 > 11$

$2x > 9$

$x > \frac{9}{2}$ ....(2)

From (1) and (2)

x can take the values 5 and 7. So the required possible paris will be (5, 7), (7, 9).

Chapter 6

PERMUTATIONS AND COMBINATIONS

GOLD COINS

- If an event can occur in 'm' different ways, following which another event can occur in 'n' different ways, then the total number occurence of the events is $m \times n$.
- $n! = n(n-1)(n-2)............3.2.1$
- $n! = n(n-1)!$
- ${}^nP_r = \frac{n!}{(n-r)!}$
- ${}^nP_n = n!$, ${}^nP_o = 1, {}^np_1 = n$
- The number of permutations of n objects where P_1 objects are one kind, P_2 are of second kind, P_3 are of third kind is $\frac{n!}{P_1!P_2!P_3!}$
- ${}^nC_r = \frac{n!}{r!(n-r)!}$,
- ${}^nC_o = 1, {}^nC_n = 1$, $nC_1 = n$, ${}^nC_r = {}^nC_{n-r}$
- If ${}^nC_a = {}^nC_b$ then either a=b or n=a+b

Golden Problems

1. Find the value of n if $^{n}P_5 = 42.{}^{n}P_3$

2. Find the value of x if $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$

3. How many 3 - digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

4. Find the number of different 8 letter arrangements that can be made from the letters of the word DAUGHTER so that

   i) all vowels occur together

   ii) all vowels do not occur together

5. If $^{n}C_8 = {}^{n}C_9$ then find $^{n}C_{17}$

6. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

7. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these (i) cards are of the same colour (ii) all are face cards (iii) 4 cards are of the same suit.

8. Find the number of permutations of the letters of the word "MISSISSIPPI".

Solutions

1. $^{n}P_5 = 42.\,^{n}P_3$

$$\frac{n!}{(n-5)!} = 42.\frac{n!}{(n-3)!}$$

$$\frac{(n-3)!}{(n-5)!} = 42$$

$$\frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 42$$

$n^2-3n-4n+12 = 42$

$n^2-7n-30 = 0$

$(n-10)\,(n+3) = 0$

$n = 10, -3$

$\therefore n = 10$

2. $$\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$$

$$\frac{10!}{8!}+\frac{10!}{9!}=\frac{10!x}{10!}$$

$10\times9+10 = x \Rightarrow x = 100$

3. Required number of 3 digit even numbers = $5\times5\times2 = 50$

4. i) Total number of words = $^{8}P_8 = 8!$

Number of words in which all vowels occur together $= {}^{6}P_6\times{}^{3}P_3$

$= 6!\ 3! = 4320$

ii) Number of words in which vowels do not occur together = $8!-4320 = 36000$

5. $n = 8 + 9 = 17$

$^{n}C_{17} = {}^{17}C_{17} = 1$

6. Number of ways = $^{6}C_3\times{}^{5}C_3\times{}^{5}C_3 = 2000$

7. Total number of ways $= {}^{52}C_4 = 270725$

i) Number of ways $= {}^{26}C_4+{}^{26}C_4 = 29900$

ii) Number of ways $= {}^{12}C_4 = 495$

iii) Number of ways $= {}^{13}C_4+{}^{13}C_4+{}^{13}C_4+{}^{13}C_4 = 2860$

8. $n=11,\ P_1=4,\ P_2=4,\ P_3=2$

Number of permutations $= \frac{n!}{P_1!P_2!P_3!} = \frac{11!}{4!4!2!} = 34650$

Chapter 7

BINOMIAL THEOREM

GOLD COINS

- $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$
- $(a+b)^n$ has n+1 terms
- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n$
- ${}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots\ldots\ldots\ldots + (-1)^n\,{}^nC_n = 0$

Golden Problems

1. Expand $\left(x+\frac{1}{x}\right)^6$

2. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$

3. Using binomial theorem find $(102)^5$

4. The number of term in the expansion of $(x+2y)^9$ is ....

5. The first term in the expansion of $(2x+3y)^5$ is ....

Solutions

1. $\left(x+\frac{1}{x}\right)^6 = {}^6C_0x^6 + {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2$

$${}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 + {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

2. $(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$(a+b)^4 - (a-b)^4 = 8a^3b + 8ab^3 = 8ab(a^2+b^2)$$

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\sqrt{3}\sqrt{2}[3+2] = 40\sqrt{6}$$

3. $(102)^5 = (100+2)^5 = {}^5C_0100^5 + {}^5C_1100^4\times 2 + {}^5C_2100^3\times 2^2$

$$+{}^5C_3100^2\times 2^3 + {}^5C_4\times 100\times 2^4 + {}^5C_5\times 2^5 = 11040808032$$

4. 10

5. $(2x)^5$ or $32x^5$

Chapter 8

SEQUENCES AND SERIES

GOLD COINS

- A sequence $a_1, a_2, a_3, \ldots\ldots\ldots a_n$ is called geometric progression, if each term is non-zero and $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}$
- If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n$ are in G.P then

  (1) n^{th} term is $a_n = ar^{n-1}$

  (2) Sum of n terms = $S_n = \frac{a(r^n - 1)}{r - 1}$
- If a, b, c are in G.P then $b^2 = ac$
- Geometric mean of a and b is $\sqrt{ab}$.

Golden Problems

1. Find the 12^{th} term of the sequence whose n^{th} term is $\frac{2n-3}{6}$
2. Find the 10^{th} term of a G.P. whose 3^{rd} term is 24 and 6^{th} term is 192.
3. The sum of first three terms of a GP is $\left(\frac{39}{10}\right)$ and their product is 1. Find the terms.
4. Insert 3 numbers between 1 and 256. So that the resulting sequence is a G.P.
5. Find the 20^{th} term of the sequence $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots\ldots\ldots$
6. Find the sum to n term of the sequence. 7+77+777+........
7. Which term of the sequence 2, $2\sqrt{2}, 4, \ldots\ldots\ldots\ldots\ldots$ is 128
8. Find the Geometric mean of 8 and 2.
9. Find the value of 'x' if $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P. ....

Solutions

1. $a_n = \frac{2n-3}{6}$

$\therefore a_{12} = \frac{2\times12-3}{6} = \frac{21}{6} = \frac{7}{2}$

2. We have $a_3 = 24 \Rightarrow ar^2 = 24$ (1)

$a_6 = 192 \Rightarrow ar^5 = 192$ (2)

from (1) and (2) $\frac{ar^5}{ar^2} = \frac{192}{24}$

$r^3 = 8$

$\Rightarrow r = 2$

$\therefore (1) \Rightarrow a\times 4 = 24$

$\therefore a = 6$

$\therefore a_{10} = a.r^9$

$= 6\times 2^9$

$= 6\times 512$

$=$ **3072**

3. Let the terms are $\frac{a}{r}$, a, ar

Given that $\frac{a}{r}\times a\times ar = 1$

$a^3 = 1 \Rightarrow a = 1$

Also given that $\frac{a}{r} + a + ar = \frac{39}{10}$

$\frac{1}{r} + 1 + r = \frac{39}{10}$ $(\because a = 1)$

$\frac{1+r+r^2}{r} = \frac{39}{10}$

$10 + 10r + 10r^2 = 39r$

$10r^2 - 39r + 10r + 10 = 0$

$10r^2 - 29r + 10 = 0$

$$\therefore \quad r = \frac{29 \pm \sqrt{29^2 - 4\times10\times10}}{2\times10}$$

$$r = \frac{29 \pm \sqrt{841 - 400}}{20}$$

$$r = \frac{29 \pm \sqrt{441}}{20}$$

$$r = \frac{29 \pm 21}{20}$$

$$r = \frac{50}{20} \quad \text{or} \quad r = \frac{8}{20}$$

$$r = \frac{5}{2} \quad \text{or} \quad \frac{2}{5}$$

when $a = 1$ & $r = \frac{5}{2}$, terms are $\frac{2}{5}$, 1, $\frac{5}{2}$

when a =1 & $r = \frac{2}{5}$, terms are $\frac{5}{2}$, 1, $\frac{2}{5}$

4. Let 1, a_2, a_3, a_4, 256 be a G.P.

$a=1$, $a_5 = 256$

$\therefore\ ar^4 = 256$

$1.r^4 = 4^4$

$\therefore\ r = 4$

$\therefore\ a_2 = 4 \quad a_3=16 \quad a_4=64$

4	256
4	64
4	16
	4

5. Let $a = \frac{5}{2}$ $\qquad r = \dfrac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{4}\times\frac{2}{5} = \frac{1}{2}$

$a_{20} = a.r^{19}$

$$= \frac{5}{2}.\left(\frac{1}{2}\right)^{19}$$

$$= 5.\left(\frac{1}{2}\right)^{20}$$

$a_n = ar^{n-1}$

6. Let Sn =7+77+777+................ to n terms

$= 7[1+11+111+ .... \text{ to n terms}]$

$= \frac{7}{9}[9+99+999+.......\text{to n terms}]$

$= \frac{7}{9}[10-1+100-1+1000-1+.......\text{to n terms}]$

$= \frac{7}{9}\left[10+10^2+10^3+............\text{to n terms}+(-1-1-1.........\text{to n terms})\right]$

$= \frac{7}{9}\left[\frac{10(10^n-1)}{9}-(1+1+1+.........\text{to n terms})\right]$

$= \frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right]$

7. $a = 2, r = \sqrt{2}, \left(\because r = \frac{2\sqrt{2}}{2} = \sqrt{2}\right)$

$a_n = ar^{n-1} = 128$

$2\left(\sqrt{2}\right)^{n-1} = 128$

Dividing both sides by 2

$\sqrt{2}^{n-1} = 64 = 2^6 = \left[\left(\sqrt{2}^2\right)\right]^6$

$\sqrt{2}^{n-1} = \sqrt{2}^{12}$

$n-1 = 12 \Rightarrow n = 13$

8. Geometric mean of a and b

$= \sqrt{ab}$

$= \sqrt{8\times 2}$

$= \sqrt{16}$

$= 4$

9. We have a, b, c in GP.

then $b = \sqrt{ac}$

$\therefore x = \sqrt{\frac{-2}{7}\times\frac{-7}{2}}$

$x = \sqrt{1}$

$x = \pm 1$

Chapter 9

STRAIGHT LINES

GOLD COINS

(a) If a line make an angle θ with positive direction of x-axis then the slope of the line (m) = tan θ

(b) Slope m of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

(c) If two lines are parallel, then their slops are equal.

(d) If two lines are perpendicular, then product of their slopes is - 1

(e) An acute angle between the lines whose slopes are m_1 and m_2 is

$$tan\,\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

(f) The line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

(g) The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(h) The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$

(i) The equation of the line making intercepts a and b on x-and y-axis, respectively, is $\frac{x}{a} + \frac{y}{b} = 1$

(j) The perpendicular distance d of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

Golden Problems

1. Find the slope of the line passing through the points (3, -2) and (-1, 4).

2. Lines through the points (-2,6) and (4, 8) is perpendicular to the line through the points (8,12) and (x, 24). Find the value of x?

3. Find the equation of the line passing through the points (1,-1) and (2, 3).

4. Find the equation of the line intersecting the x-axis at a distance of 3 units left of origin with slope-2.

5. Reduce the following equation into interept form, and find their interepts on the axes.

   $3x+2y-12 = 0$

6. Reduce into slope intercept form and find its slope and y-intercept of the line.

   $6x+3y-5 = 0$

7. Find the distance of the point (-1, 1) from the line $3x-4y+5=0$.

8. Find the angle between the lines.

   $y-\sqrt{3}x-5=0$ and $\sqrt{3}y-x+6=0$

9. Find the equation of a line perpendicular to the line $x-2y+3=0$ and passing through the point (1, -2).

Solutions

1. We have slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\therefore\ m = \frac{4+2}{^-1-3}$$

$$= \frac{6}{^-4}$$

$$= \frac{^-3}{2}$$

2. We have, two lines are perpendicular, then the product of their slopes is -1.

ie; m_1 is the slope of first line

m_2 is the slope of second line

then $m_1 \times m_2 = -1$

Now $m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

But $m_1\, m_2 = -1$

$$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$$

$$-4 = x - 8$$

$$\therefore x = 8-4$$

$$x = 4$$

3. We have equation of the line passing through two given points is

$$(y-y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\therefore \quad (y-^-1) = \frac{3-^-1}{2-1}(x-1)$$

$$y+1 = \frac{4}{1}(x-1)$$

$y = 4x-4-1$

$y = 4x-5$

4. Equation of the line when one point and slope are given is

$$(y-y_1) = m(x-x_1) \qquad (1)$$

Here m = -2 & (x_1, y_1)= (-3, 0)

$\therefore$ (1) becomes $(y-0) = -2(x--3)$

$y = -2(x+3)$

$y = -2x-6$

$2x+y+6 = 0$

5. We have interept form is $\frac{x}{a}+\frac{y}{b}=1$

$3x+2y-12 = 0$

$\Rightarrow \quad 3x+2y = 12$

$\frac{3x}{12}+\frac{2y}{12} = 1$

$\frac{x}{4}+\frac{y}{6} = 1$

$\therefore$ x-intercept = 4

y-intercept = 6

6. We have slope intercept form is

$y = mx+c$

$6x+3y-5 = 0$

$\Rightarrow \quad 3y = -6x+5$

$y = \frac{-6}{3}x+\frac{5}{3}$

$y = -2x+\frac{5}{3}$

$\therefore$ Slope= -2 & y-interpret $=\frac{5}{3}$

7. We have distance of a point (x_1, y_2) from the line $ax+by+c=0$ is $\left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$

$$\therefore \text{distance} = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right| \qquad (1)$$

Here $(x_1, y_1) = (-1, 1)$ A=3 B = -4 C = 5

Substitute these values in (1), we get

$$\text{Distance} = \left|\frac{3\times{}^-1 + {}^-4\times 1 + 5}{\sqrt{3^2 + ({}^-4)^2}}\right|$$

$$= \left|\frac{{}^-3 - 4 + 5}{\sqrt{9+16}}\right|$$

$$= \left|\frac{{}^-7+5}{\sqrt{25}}\right| = \left|\frac{-2}{5}\right| = \frac{2}{5} \text{ units}$$

8. We have angle between the lines whose slopes one known is

$$\tan\theta = \left|\frac{m_2 - m_1}{1+m_1m_2}\right| \qquad (1)$$

Where m_1 & m_2 are slopes of first and second lines respectively.

$y-\sqrt{3}x-5=0$ and $\sqrt{3}y-x+6=0$

$\Rightarrow y = \sqrt{3}x+5$ $\qquad \sqrt{3}y = x-6$

$m_1=\sqrt{3}$ $\qquad y = \frac{1}{\sqrt{3}}x - \frac{6}{\sqrt{3}}$

$m_2 = \frac{1}{\sqrt{3}}$

Now (1) $\Rightarrow$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}\right| = \left|\frac{\frac{1-\sqrt{3}\times\sqrt{3}}{\sqrt{3}}}{1+1}\right|$$

$$= \left|\frac{1-3}{2\sqrt{3}}\right|$$

$$= \left|\frac{-2}{2\sqrt{3}}\right|$$

$$= \left|\frac{-1}{\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \therefore \ \theta = 30^\circ \ \text{or} \ \theta = 150^\circ$$

9. We have equation of the line is

$(y-y_1) = m(x-x_1)$ (1)

Let

$$x-2y+3 = 0$$

$$2y = x+3$$

$$y = \frac{1}{3}x+1$$

$\Rightarrow$ slope of the line

$= \lambda = \frac{1}{3}$ also the point is (1, -2).

$\therefore$ Slope of the line $\perp^r$ to the given line is $\frac{-1}{\frac{1}{3}} = -3$

$$\therefore (1) \Rightarrow (y - {}^-2) = -3(x-1)$$

$$y+2 = -3(x-1)$$

$$y+2 = -3x+3$$

$$3x+y-1 = 0$$

Chapter 10

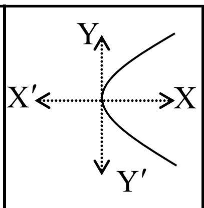
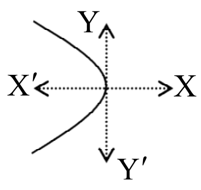
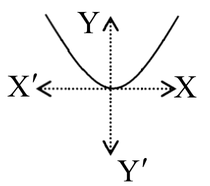
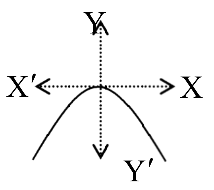
CONIC SECTIONS

GOLD COINS

Circle

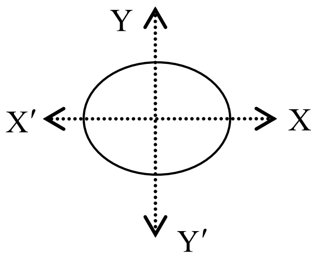
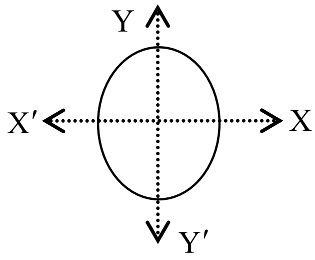
- The equation of a circle with centre (h, k) and the radius r is $(x-h)^2+(y-k)^2=r^2$
- The equation of a circle with centre (0,0) and the radius r is $x^2+y^2=r^2$
- General equation of a circle is $x^2+y^2+2gx+2fy+c=0$ centre = $(-g,-f)$ radius = $\sqrt{g^2+f^2-c}$

Parabola

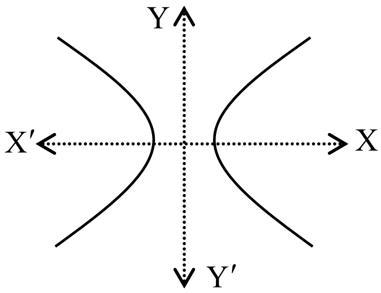
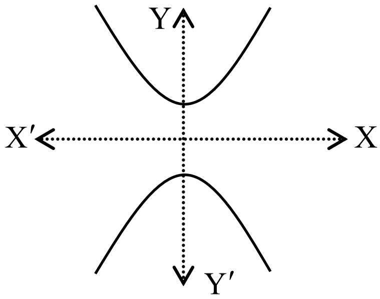
Parabolas				
Standard equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Focus	(a, 0)	(-a, 0)	(0, a)	(0, -a)
Length of latus rectum	4a	4a	4a	4a
Equation of directrix	$x+a=0$	$x-a=0$	$y+a=0$	$y-a=0$
Axis	$y=0$	$y=0$	$x=0$	$x=0$

GOLD COINS

Ellipse

Ellipse		
Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{y^2}{b^2}+\frac{x^2}{a^2}=1$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of major axis	2a	2a
Length of minor axis	2b	2b
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Eccentricity (e)	$\frac{c}{a}$	$\frac{c}{a}$

Hyperbola

Hyperbola		
Standard equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of transvers axis	2a	2a
Length of conjugate axis	2b	2b
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Eccentricity (e)	$\frac{c}{a}$	$\frac{c}{a}$

Golden Problems

1. Find the centre and radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$.

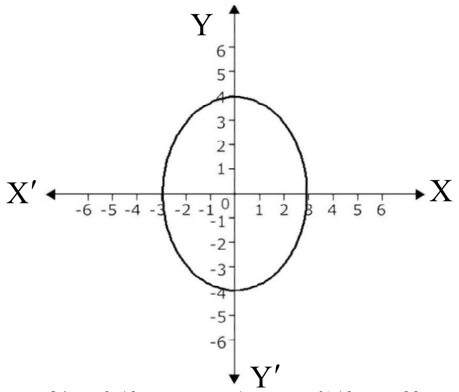
2. Find the equation of the circle with centre (1, 1) and radius 2.

3. Consider the parabola $y^2 = 8x$, find
   a. Focus
   b. Length of latus rectum
   c. Equation of the directrix

4. Find the equation of the parabola with focus (6, 0) and equation of the directrix is $x = -6$.

5. Consider the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, find
   a. Vertices
   b. Foci
   c. Eccentricity
   d. Length of latus rectum
   e. Length of major and minor axis

6. Consider the ellipse



   a) Find the equation of the ellipse
   b) Find the coordinates of the foci

7. Consider the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ find
   a. Vertices
   b. Foci
   c. Eccentricity
   d. Length of latus rectum
   e. Length of transverse and conjugate axes

Solutions

1. Given that $x^2+y^2+8x+10y-8=0$

$(x^2+8x+16)+(y^2+10y+25)=8+16+25$

$\therefore\ (x+4)^2+(y+5)^2=49$

$(x-(-4))^2+(y-(-5))^2=7^2$

It is of the form $(x-h)^2+(y-k)^2=r^2$

$\therefore$ Centre, (h, k) = (-4, -5) and Radius (r) = 7

> $(x-h)^2+(y-k)^2=r^2$
> Centre $=(h,k)$
> Radius $=r$

2. Centre, (h, k) = (1, 1) and Radius, r = 2

Equation of the circle is $(x-h)^2+(y-k)^2=r^2$

i.e., $(x-1)^2+(y-1)^2=2^2$

$x^2-2x+1+y^2-2y+1=4$

$x^2+y^2-2x-2y-2=0$

3. Given that $y^2=8x$ ⟶ (1)

It is of the form $y^2=4ax$ ⟶ (2)

From (1) and (2) we get $4a=8\ \therefore\ a=\frac{8}{4}=2$

a. Focus (a, 0) = (2, 0)
b. Length of latus rectum, 4a = 8
c. Equation of the directrix $x+a=0$
i.e., $x+2=0$

> $y^2=4ax$
> Focus = (a, 0)
> Length of latus rectum = 4a
> Equation of directrix $x+a=0$

4. Focus (a, 0) = (6, 0)

Equation of the parabola is $y^2=4ax$

$y^2=4\times 6x$

$y^2=24x$

> Equation of the parabola is
> $y^2=4ax$

5. Given that $\frac{x^2}{25}+\frac{y^2}{9}=1$

Which is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$a^2=25\Rightarrow a=5$

$b^2=9\Rightarrow b=3$

$c^2=a^2-b^2=25-9=16\Rightarrow c=4$

a) Vertices $= (\pm a, 0) = (\pm 5, 0)$

b) Foci $= (\pm c, 0) = (\pm 4, 0)$

c) Eccentricity, $= e = \frac{c}{a} = \frac{4}{5}$

d) Length of latus rectum, $\frac{2b^2}{a} = \frac{2\times 9}{5} = \frac{18}{5}$

e) Lenth of major axis, 2a = $2\times 5 = 10$
Lenth of minor axis, 2b = $2\times 3 = 6$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Vertices $= (\pm a, 0)$
Foci $= (\pm c, 0)$
Eccentricity, $e = \frac{c}{a}$
Length of latus rectum $= \frac{2b^2}{a}$
Lenth of major axis $= 2a$
Lenth of minor axis $= 2b$
$c^2 = a^2 - b^2$

6. a) From the figure $a = 4$ and $b = 3$

Equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

b) $c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

Foci, $(0, \pm c) = (0, \pm\sqrt{7})$

7. Given that

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

It is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then

$a^2 = 16 \Rightarrow a = 4$

$b^2 = 9 \Rightarrow b = 3$

$c^2 = a^2 + b^2 = 16 + 9 = 25 \Rightarrow c = 5$

a) Vertices $= (\pm a, 0) = (\pm 4, 0)$

b) Foci $= (\pm c, 0) = (\pm 5, 0)$

c) Eccentricity, $= e = \frac{c}{a} = \frac{5}{4}$

d) Length of latus rectum, $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

e) Length of transverse axis, = 2a = $2\times 4 = 8$
Length of conjugate axis, = 2b = $2\times 3 = 6$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Vertices $= (\pm a, 0)$
Foci $= (\pm c, 0)$
Eccentricity, $e = \frac{c}{a}$
Length of latus rectum $= \frac{2b^2}{a}$
Lenth of transverse axis $= 2a$
Lenth of conjugate axis $= 2b$
$c^2 = a^2 + b^2$

Chapter 11

INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

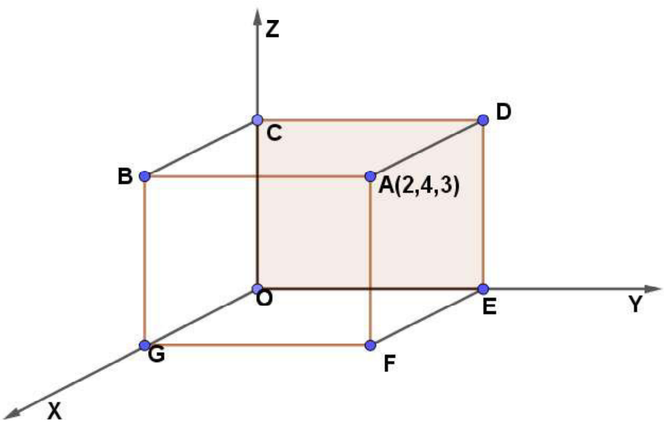
GOLD COINS

- Any point on space is of the form (x, y, z)
- Any point on XY plane is of the form $(x, y, 0)$
- Any point on YZ plane is of the form $(0, y, z)$
- Any point on XZ plane is of the form $(x, 0, z)$
- Any point on x-axis is of the form $(x, 0, 0)$
- Any point on y-axis is of the form $(0, y, 0)$
- Any point on z-axis is of the form $(0, 0, z)$
- Distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Octants	Sign of Coordinates (x, y, z)	Example
I	+ + +	$(2, 3, 4)$
II	− + +	$(-2, 3, 4)$
III	− − +	$(-2, -3, 4)$
IV	+ − +	$(2, -3, 4)$
V	+ + −	$(2, 3, -4)$
VI	− + −	$(-2, 3, -4)$
VII	− − −	$(-2, -3, -4)$
VIII	+ − −	$(2, -3, -4)$

Golden Questions

1. A point is on the x-axis. What are its y-coordinate and z-coordinate?
2. A point is in the XY-plane. What can you say about its z-coordinate?
3. Name the octants in which the following points lie
   a. (1, 2, 3)
   b. (4, –2, 3)
   c. (4, –2, –5)
   d. (4, 2, –5)
   e. (– 4, 2, –5)
   f. (– 4, 2, 5)
   g. (– 2, – 4, –7)
4. Find the distance between the points A (2, 3, 5) and B (4, 3, 1).
5. Show that the points A(0,7, -10), B(1, 6, -6) and C(4, 9, -6) are the vertices of an isosceles triangle.
6. Show that the points A(-2, 3, 5) , B $(1, 2, 3)$ and C $(7, 0, -1)$ are collinear.
7. Consider the following figure



a) In which plane the point B lies ?
b) Write the coordinates of B.
c) Find the distance AB.

Solutions

1. y-coordinate = 0 and z - coordinate = 0
2. z - coordinate = 0
3. a. I

   b. IV

   c. VIII

   d. V

   e. VI

   f. II

   g. VII

4. $AB = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

   $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

   $= \sqrt{2^2 + 0^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

5. $AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1^2 + (-1)^2 + 4^2} = \sqrt{18}$

   $BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{3^2 + 3^2 + 0^2} = \sqrt{18}$

   Thus *AB=BC*. Hence ΔABC is an isosceles triangle

6. $AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{3^2 + (-1)^2 + (-2)^2} = \sqrt{14}$

   $BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{6^2 + (-2)^2 + (-4)^2} = \sqrt{56} = 2\sqrt{14}$

   $AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{9^2 + (-3)^2 + (-6)^2} = \sqrt{126} = 3\sqrt{14}$

   Thus AB + BC = AC. Hence the points A, B, C are collinear.

7. a) The point B lies in XZ plane

   b) B = (2, 0, 3)

   c) AB = 4

Chapter 12

LIMITS AND DERIVATIVES

GOLD COINS

If f(x) is a polynomial function then $\lim_{x\to a} f(x) = f(a)$

- $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x)$
- $\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}, \lim_{x\to 0} \frac{\text{Sin}\, x}{x} = 1$
- $f^1(x) = \lim_{h\to o} \frac{f(x+h) - f(x)}{h}$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$
- $\frac{d}{dx}(\sin x) = \text{Cos}\, x$
- $\frac{d}{dx}(\cos x) = {}^-\text{Sin}\, x$
- $\frac{d}{dx}(\tan x) = \text{Sec}^2\, x$
- $\frac{d}{dx}(\cot x) = -\text{Cos}\,ec^2 x$
- $\frac{d}{dx}(\sec x) = \text{Sec}\, x.\tan x$
- $\frac{d}{dx}(\text{Cosecx}) = -\text{Cosecx}.C\,\text{ot}\, x$

GOLD COINS ...

- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}[Kf(x)] = K\frac{d}{dx}[f(x)]$, k is a constant
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$
- $\frac{d}{dx}[f(x).g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$

Golden Problems

1 Find the derivative of f(x) = Sin x using first principle.

2 Find the derivative of $\frac{x^5 - \text{Cos}\, x}{\text{Sin}\, x}$

3 Evaluate $\lim_{x \to 0} \frac{\text{Sin}\, 3x}{\text{Sin}\, 4x}$

4. Evaluate $\lim_{x \to 0} \frac{x^{15} - 1}{x^{10} - 1}$

5 Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

6 Differentiate $\frac{\text{Cos}\, x}{1 + \text{Sin}\, x}$ with respect to x.

7 Find the derivative of f(x) = $\frac{1}{x}$ using first principle.

8 f(x) = $2x^3$-1, then find $f^1(1)$

9 Find the derivative of $(x^2 + 1)\text{Cos}\, x$

10 Find $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

11 If f(x) = $\begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, \ x = 0 \end{cases}$ then find $\lim_{x \to 0} f(x)$

12 Find derivative of x.Sin x

13 If f(x) = 1+x+x^2+x^3+............+ x^{50} then find $f'(1)$

14 $\frac{d}{dx}\left(2x - \frac{3}{4}\right) = $ ....

Solutions

1 $$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\text{Sin}(x+h)-\text{Sin}\,x}{h}$$

$$= \lim_{h\to 0}\frac{2\,\text{Cos}\left(\frac{2x+h}{2}\right)\text{Sin}\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h\to 0}\text{Cos}\left(x+\frac{h}{2}\right)\lim_{h\to 0}\frac{\text{Sin}\left(h/2\right)}{h/2}$$

$$= \text{Cos}\,x$$

2 $$\frac{d}{dx}\left(\frac{x^5-\cos x}{\sin x}\right) = \frac{\sin x\frac{d}{dx}(x^5-\cos x)-(x^5-\cos x)\frac{d}{dx}\sin x}{(\sin x)^2}$$

$$= \frac{\sin x(5x^4+\sin x)-(x^5-\cos x)\cos x}{\sin^2 x}$$

3. $$\lim_{x\to 0}\frac{\sin 3x}{\sin 4x} = \lim_{x\to 0}\frac{\frac{\sin 3x}{3x}\times 3x}{\frac{\sin 4x}{4x}\times 4x}$$

$$= \frac{3}{4}\frac{\lim_{x\to 0}\frac{\sin 3x}{3x}}{\lim_{x\to 0}\frac{\sin 4x}{4x}} = \frac{3}{4}$$

4. $$\lim_{x\to 0}\frac{x^{15}-1}{x^{10}-1} = \lim_{x\to 0}\frac{\frac{x^{15}-1^{15}}{x-1}\times x-1}{\frac{x^{10}-1^{10}}{x-1}\times x-1}$$

$$= \frac{15\times 1^{14}}{10\times 1^{9}} = \frac{15}{10} = \frac{3}{2}$$

5. $$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

Put $1+x=y$

$x=y-1$

$x \to 0, y \to 1$

$$= \lim_{y \to 1} \frac{\sqrt{y}-1}{y-1}$$

$$= \lim_{y \to 1} \frac{y^{1/2}-1^{1/2}}{y-1} = \frac{1}{2}$$

6. $$\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right) = \frac{(1+\sin x)\frac{d}{x}\cos x - \cos x \frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x) - \sin x - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2} = \frac{-1}{1+\sin x}$$

7. $$f^1(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x-(x+h)}{x(x+h)h} = \lim_{h \to 0} \frac{-h}{x(x+h)h}$$

$$= -\frac{1}{x^2}$$

8. $f(x) = 2x^3-1$

$f^1(x) = 6x^2$

$f^1(1) = 6$

9. $$\frac{d}{dx}(x^2+1)\text{Cos}\, x = (x^2+1)\times {}^-\sin x + \cos x(2x)$$

$$= -x^2 \sin x - \sin x + 2x\cos x$$

10. $$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2}$$

$$\lim_{x \to 2} x+2 = 4$$

11. $f(x)=\begin{cases}\dfrac{|x|}{x}, x\neq 0\\ 0, x=0\end{cases}$

$f(x)=\begin{cases}-1, x<0\\ 0, x=0\\ 1, x>0\end{cases}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} -1 = -1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 1 = 1$

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore$ Limit doesn't exist.

12. $\frac{d}{dx}(x\sin x) = x\cos x + \sin x.1$

13. $f(x) = 1+x+x^2+x^3+.............+x^{50}$

$f^1(x) = 0+1+2x+3x^2+4x^3+.............+50x^{49}$

$f^1(1) = 1+2+3+4+........+50$

$= \frac{50\times 51}{2} = 1275$

14. $\frac{d}{dx}\left(2x-\frac{3}{4}\right) = \frac{d}{dx}(2x)-\frac{d}{dx}\left(\frac{3}{4}\right)$

$= 2$

Chapter 13

STATISTICS

GOLD COINS

- **For grouped data**

Mean ($\bar{x}$) = $\dfrac{\sum_{i=1}^{n} f_i x_i}{N}$, where $N = \sum_{i=1}^{n} f_i$

Variance = $\dfrac{\sum_{i=1}^{n} f_i x_i^2}{N} - (\bar{x})^2$

Standard deviation = $\sqrt{\text{variance}}$

Mean deviation about mean = $\dfrac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$ (where $\bar{x}$ is the mean)

Mean deviation about median = $\dfrac{1}{N}\sum_{i=1}^{n} f_i |x_i - M|$ (Where M is the Median)

- **For ungrouped data**

Mean ($\bar{x}$) = $\dfrac{\sum_{i=1}^{n} x_i}{n}$, n = number of observations

Variance (σ^2) = $\dfrac{\sum_{i=1}^{n} x^2}{n} - (\bar{x})^2$

Standard deviation = $\sqrt{\text{variance}}$

Mean deviation about mean = $\dfrac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$ (where $\bar{x}$ is the mean)

Mean deviation about median = $\dfrac{\sum_{i=1}^{n} |x_i - M|}{n}$ (Where M is the Median)

Coefficient of variation = $\dfrac{\sigma}{\bar{x}} \times 100$

Golden Problems

1. Find mean, variance, SD and coefficient of variation for the observations 2, 4, 6, 8, 10

2. Find mean, variance and standard deviation for the following data.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

3. Find mean deviation about mean for

x	2	5	6	8	10	12
f	2	8	10	7	8	5

4. Find mean deviation about median for

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No.of students	6	8	14	16	4	2

5. Find mean deviation about mean for

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	3	8	14	8	3	2

6. Find mean, variance and SD for

x	3	8	13	18	23
f	7	10	15	10	6

Solutions

1. n=5, $\sum x = 2+4+6+8+10=30$

$\sum x^2 = 4+16+36+64+100=220$

Mean $= \bar{x} = \frac{\sum x}{n} = \frac{30}{5} = 6$

Variance $= \sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

$= \frac{220}{5} - 36$

$= 44-36$

$= \mathbf{8}$

SD (σ) $= \sqrt{\text{variance}} = \sqrt{8} = \mathbf{2.83}$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{2.83}{6} \times 100$

$= \mathbf{47.17}$

2.

Class	f	x	fx	fx^2
0-10	5	5	25	125
10-20	8	15	120	1800
20-30	15	25	375	9375
30-40	16	35	560	19600
40-50	6	45	270	12150
	50		**1350**	**43050**

N $= \sum f$ **= 50**

Mean = $\frac{\sum fx}{N} = \frac{1350}{50} = 27$

Variance (σ^2) $= \dfrac{\sum fx^2}{N} - (\overline{x})^2$

$= \dfrac{43050}{50} - 729$

$= 861 - 729$

$=$ **132**

SD (σ) $= \sqrt{132}$

= 11.49

3.

x	f	fx	$\|x-\overline{x}\|$	$f\|x-\overline{x}\|$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
N	40	300		92

N = 40

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$$

$= \dfrac{300}{40}$ **=7.5**

Mean deviation about mean $= MD(\overline{x})$

$$= \frac{\sum_{i=1}^{n} f_i |x_i - \overline{x}|}{N}$$

$= \dfrac{92}{40}$ **= 2.3**

4.

Class	f	c.f	x	$\|x_i-M\|$	$f_i\|x_i-M\|$
0-10	6	6	5	22.857	137.142
10-20	8	14	15	12.857	102.856
20-30	14	28	25	2.857	39.998
30-40	16	44	35	7.143	114.288
40-50	4	48	45	17.143	68.572
50-60	2	50	55	27.143	54.286
	N =50				**517.142**

$\frac{N}{2}$ =25, median class = 20-30

$\ell = 20$, f = 14, c = 14, h = 10

Median $= \ell + \frac{\left(\frac{N}{2} - c\right)h}{f}$

$= 20 + \frac{11 \times 10}{14}$

$=$ **27.857**

MD(M) $= \sum_{i=1}^{n} f_i \,|x_i - M| = \frac{517.142}{50} =$ **10.34**

5.

Class	f	x	fx	$\|x-\bar{x}\|$	$f\|x-\bar{x}\|$
10-20	2	15	30	30	60
20-30	3	25	75	20	60
30-40	8	35	280	10	80
40-50	14	45	630	0	0
50-60	8	55	440	10	80
60-70	3	65	195	20	60
70-80	2	75	150	30	60
Total	**40**		**1800**		**400**

$$N = \sum_{i=1}^{n} f_i = 40$$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{1800}{40} = 45$$

$$MD(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$$

$$= \frac{400}{40} = 10$$

6.

x	f	fx	fx^2
3	7	21	63
8	10	80	640
13	15	195	2535
18	10	180	3240
23	6	138	3174
	48	614	9652

$$N = \sum_{i=1}^{n} f_i = 48$$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$$

$$= \frac{614}{48}$$

=12.79

$$\text{Variance} = \frac{\sum_{i=1}^{n} f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{9652}{48} - (12.79)^2 = 201.08 - 163.58$$

$$= 37.5$$

SD = **6.12**

Chapter 14

PROBABILITY

GOLD COINS

If A and B are two events then,

1. $P(A) = \frac{n(A)}{n(S)} = \frac{\text{Number of cases favourable to A}}{\text{Total possible outcomes}}$

2. $0 \le P(A) \le 1$

3. $P(\phi) = 0$

4. $P(S) = 1$

5. $P(A') = 1 - P(A)$

6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

7. P[A and B] = $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

8. P[A and not B] = $P(A \cap B') = P(A) - P(A \cap B)$

9. P(not A) = $P(A')$

10. P(A or B) = $P(A \cup B)$

11. P(not A and not B) = $P(A' \cap B')$

Golden Problems

1. $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find

   i) $P(A \cup B)$ ii) $P(A' \cup B')$ iii) $P(A \cap B')$

2. Three coins are tossed once. Find probability of getting i) 3 heads, ii) at least 2 heads iii) exactly 2 heads.

3. One card is drawn from a pack of 52 playing cards. Find probabilities that the card will be

   i) a diamond ii) a king iii) a red

4. A committee of 2 persons is selected from 2 men and 2 women. What is the probability that the committee will have i) one man, ii) two men.

5. Two dice are thrown. Find the probability of getting

   i) a doublet

   ii) sum of the numbers on the dice is 6

   iii) sum of the numbers on the dice ≤ 4.

6. $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$, $P(A \cap B) = \frac{1}{15}$

   Find

   i) P (not A)

   ii) P (A or B)

   iii) P(A and not B)

   iv) P (not A and not B)

Solutions

1. i) $$P(A \cup B) = P(A) + P(B) = P(A \cap B)$$
$$= 0.54 + 0.69 - 0.35$$
$$= 0.88$$

ii) $$P(A' \cup B') = P(A \cap B)'$$
$$= 1 - P(A \cap B)$$
$$= 1 - 0.35$$
$$= 0.65$$

iii) $$P(A \cap B') = P(A) - P(A \cap B)$$
$$= 0.54 - 0.35$$
$$= 0.19$$

2. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

i) $A = \{HHH\}$
$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) $B = \{HHH, HHT, HTH, THH\}$
$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

iii) $C = \{HHT, HTH, THH\}$
$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$

3. $n(S) = {}^{52}C_1 = 52$

i) $$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

ii) $$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

iii) $$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

4. $n(S) = 4C_2 = 6$

i) P(one man)= $\frac{2C_1 \times 2C_1}{6} = \frac{4}{6} = \frac{2}{3}$

ii) P(2 men) $= \frac{2C_2}{6} = \frac{1}{6}$

5. S = { (1, 1), (1, 2), ...(1,6)

(2, 1), (2, 2), ...(2,6)

(3, 1), (3, 2), ...(3, 6)

....

....

(6, 1), (6, 2), ...(6, 6) }

n(S) = 36

i) A = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

ii) B = { (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) }

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

iii) C = { (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}

n(C) = 6

$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

6. i) P (not A) $= P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

ii) P (A or B) $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15}$

$= \frac{8}{15} - \frac{1}{15}$

$= \frac{7}{15}$

iii) P(A and not B) $= P(A \cap B')$

$= P(A) - P(A \cap B)$

$= \frac{1}{3} - \frac{1}{15} = \frac{4}{15}$

iv) P (not A and not B) $= P(A' \cap B')$

$= P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - \frac{7}{15} = \frac{8}{15}$